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Experimental "shakedown" of continuous steel beams

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EXPERIMENTAL "SHAKEDOWN" OF CONTINUOUS STEEL BEAMS

by

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This investigation has been carried out in fulfillment of requirements for

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EXPERIMENTAL "SHAKEDOWN" OF CONTINUOUS STEEL BEAMS

Introduction

When the plastic behavior of a statically indeterminate structure at failure is considered, it is commonly associated with the application of a proportional steadily increasing system of loads. However, when the loads within this sytem are varied within certain limits either independently of each other or in a certain loading pattern, then it is possible that the structure will fail upon repeated application of this loading pattern. As stated by Symonds[(1)*], this type of failure is considered in this report.

It was desired to investigate experimentally the behavior of a statically indeterminate structure under the conditions of proportional loading and repeated variable loading. The structure so chosen was a continuous beam, simply supported over two equal spans and carrying two concentrated loads at points symmetrical about the central support (Figure 1). A theoretical consideration of the structure is presented using some recent methods of plastic analysis, and an attempt is made to explain the continued deformation of the structure under the application of a certain cyclic load. The choice of the cyclic loading steps together with the basis of comparison of test results are also presented as part of this report.

Massonnet[(2)] carried out tests on a structure under loads applied and removed in random manner and had great difficulty in achieving results due to lateral buckling. Neal[(3)],

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* See list of references at end of report.

in his paper, summarized the work of previous investigators on the shakedown theory of trusses, He also stated his corresponding proof in the case of frame structures(4):-

> "If any state of residual stress can be found for a structure that enables all further variations of the external loads between their prescribed limits to be supported in a purely elastic manner, then the structure will shakedown."

The experimental determination of the critical shakedown load, the failure load for a two span continuous beam and the comparison with values predicted by theory was the main object of this investigation.

Theoretical Analysis

(a) Proportional Loading:

In the structure shown in Figure 1, the points of support and load application are numbered 1, 2, 3, 4, and 5. Shear being constant between these points, the bending moment will accordingly vary linearly, making possible the formation of plastic hinges as points 2, 3, and 4, the points of maximum and minimum bending moment. If the values of bending moment at points 2 to 4 are known the shape of the bending moment diagram can immediately be determined. The convention adopted in this analysis is that tension on the bottom fibre of the beam is caused by a positive bending moment.

From the figure it can be seen that two possible loading conditions may produce collapse of the structure. In case

(a) two equal loads, symmetrically placed are acting on adjacent spans (Figure 1) and in case (b) a single concentrated load is applied to one span only (Figure 2). The collapse mechanism for both systems of loading give identical collapse loads. It can be shown that the collapse load P_c is

$$P_c = 20M_p/3L \qquad (1)$$

where M_p is the full plastic moment of the section.

It is interesting to note that although both cases have the same numerical collapse load value the sequence of formation of the plastic hinges is reversed. Likewise, the deflections of the load point at collapse are not the same in both cases. Simple plastic theory[6] predicts load-deflection curves as shown in Figure 3.

(b) Cyclic Loading:

The structure being symmetrical, deformations of joints 2 and 3 will only be analyzed. When the structure is subjected to independently varying loads beyond certain critical limits, none of which would produce simultaneous formation of plastic hinges, definite rotations (in the same sense) can be built up at these joints. If loads as shown in Figures 1 and 2 compose the loading cycle, case (a) rotates hinge 3 while (b) rotates hinge 2. As a result the deflection at point 2 is increased at the end of each cycle. Such cyclic repetitions will eventually produce excessive deflections.

The continued deflection as each cycle is applied can only occur in a statically indeterminate structure wherein residual moments exist as a consequence of plastic deformations at the joints. An example of this occurs in case (a) Figure 1 when the beam is loaded as shown and joint 3 deforms plastically. The equal loads are subsequently removed and under this zero load condition, joint 3 produces an effect which would deflect point 1 downward were the support at this point removed. There exists then, a positive residual moment on the beam in equilibrium with the zero loads. The loading as in case (b) is then applied and when the sum of the positive residual moment and the superimposed bending moment at joint 2 is greater than the available plastic moment of the section, the deflection will increase a finite amount. Joint 2 may behave in similar fashion for the loading sequence case (b) to case (a) further increasing the deflection.

After sufficient applications of the cyclic load, the structure may have acquired a particular set of residual moments whereby all further applications of the prescribed loads will be supported in an elastic manner. This prescribed load is the "shakedown load" above which deflections will continue to increase resulting in excessive deformations of the structure.

Due to conditions inherent to the chosen test set-up, a single concentrated load necessary for maximum bending moment at joint 2 could not be realized exactly but was accompanied by a one kip load acting on the adjacent span as shown in Figure 4(b). Maximum moment for joint 3 occurred for the loading condition shown in Figure 4(a). Using any classical method of analysis, the elastic bending moment diagrams for such critical loading conditions can be determined. These are shown in Figure 4(a) and 4(b). The residual moment diagram can only have the shape shown in Figure 4(c) the sign of the moment being chosen arbitrarily. Based on the necessary condition for the shakedown theory - the sum of the residual moment and the superimposed bending moment must not exceed the plastic moment of the section - the following inequalities for joints 2 and 3 respectively must hold:

$$\frac{P_s\ L}{625}(114 - 36\xi) + \frac{3}{5}M_r \leqslant M_p \qquad - - - (2)$$

$$-\frac{120}{625}P_s\ L + M_r \geqslant -M_p \qquad - - - - (3)$$

M_r is the residual moment under zero load at joint 3, P_s being the shakedown load and ξ a numerical coefficient less than unity. Solving equations (2) and (3) gives:

$$P_s \leqslant \frac{1000}{186-36}\left(\frac{M_p}{L}\right) \qquad - - - - - - - - - (4)$$

For alternating plastic flow not to occur at joint 2, a second necessary condition requires that the sum of the maximum and minimum bending moment must not exceed the available elastic moment range of the cross-section ΔM_y:

$$\frac{P_s\ L}{625}(114 - 36\xi) + \frac{P_s\ L}{625}(36 - 114\xi) \leqslant \Delta M_y \quad - -(5)$$

Simplifying eq. (5):

$$\frac{6}{25}P_s\ L\ (1 - \xi) \leqslant \Delta M_y \qquad - - - - - - (5a)$$

Experimental Investigation

1. Test Set-up

The test set-up shown in Figure 5(a) and (b) is a simple indeterminate structure, i.e. a continuous beam simply supported over two equal spans and carrying two concentrated loads at points symmetrical about the central support. The two concentrated loads at points 2 and 4 (Figure 1) were applied by means of hydraulic jacks and measured by calibrated dynamometers. A frame system was devised such that the jacks were acting in tension as were the supports of the beam.

To simulate simple supports, thin plates (Figure 6 and 7) were installed whose moment of inertia as compared with that of the specimen was in the ratio of approximately 1/250 at the critical central support. SR-4 gages (Figure 6) were attached at both end supports to measure the end reactions of the beam.

Ames dials were used throughout the test program for the measurement of deflections and joint rotations. Deflection readings were taken at the two load points and possible movement of the supports was checked from time to time as the test progressed. Rotation was measured at joints 2, 3, and 4 with indicators installed over a length equal to the depth of the section.

A 4WF13, as-delivered taken from the middle third position of the same rolling, was used. A span length of 4 feet was considered sufficient for this program. The loading stiffeners together with the support stiffeners were welded to the specimens prior to stress relieving treatment at the Bethlehem Steel Plant. Thus all specimens were practically free of any residual stress due to cold bending and welding.

Since tests were to be carried far into the plastic range, effects of lateral buckling were eliminated by testing the specimen about the weak axis.

The ratio of the collapse load to the shakedown load of the structure increases as the point of application of the load approaches the central support. As will be shown later, it was desired that this ratio be not less than 120% to compensate for any experimental error. Furthermore, a not too steep moment gradient was sought. Accordingly the load point was selected at a distance of 2/5 of the span length from the central support.

The loading jacks were properly plumbed and aligned with the axis of the specimen.

Though the beams were annealed, they were whitewashed in order to make a qualitative study of the yielding process. The plate supports were also whitewashed such that possible yield lines due to stress concentration could have been observed.

2. Test Program

The tests performed in this investigation are summarized in Table 1. Two collapse tests were performed under proportional loading with the load applied in sufficient increments determined from a load-deflection graph plotted during the test. Due to test set-up conditions, a zero load on a span was replaced by a 1 kip load. In the inelastic range, readings were taken when the load and deformations were fairly stable.

In test P-2, the beam was unloaded at 17.7 kips and loaded again till the deformations were excessive. This was necessary in order to verify the residual moment pattern and magnitude. As a basis of comparison, test P-2 was used as a control for the cyclic loading tests. Again test P-1 was used to check the collapse behavior of test P-2 as their predicted collapse loads were identical.

Three cyclic loading tests (sometimes referred to as tests of progressive collapse) were deemed appropriate. As mentioned previously in this report, only the load necessary to produce maximum bending moment at the joints 2, 3, and 4 composed the cyclic load. Shown in Table 1 are the steps that constituted one loading cycle. The procedure followed for starting the cyclic test was to bring both equal-loads to the applied shakedown value corresponding to Step (a), with readings taken at appropriate increments. Thereafter, readings were taken only after each step was completed. Step (e) concluded one cycle. Under these circumstances, the deformation at stage (e) was the basis of comparison for purposes of this paper.

A sufficient number of loading cycles was applied till deformations stabilized. On the two tests C-2 and C-3, the applied shakedown load was increased each time the structure stabilized, thus reducing the number of specimens required to determine the experimental critical shakedown load. Test C-1 was to represent a typical progressive collapse behavior.

Four representative tension coupons were tested in a 60,000# hydraulic machine under laboratory strain rate[7], with the use of a Templin stress-strain recorder on an 8" gage length. The tension coupons were dimensioned according to ASTM standards. Similarly the geometrical properties of the section were obtained accurately using micrometers, and these were checked against carbon imprints.

3. Test Results

Shown in Table 2 is a summary of properties of the 4WF13 shape, tested in this program. Also included are the section properties derived from the material and geometrical properties. With this table the predicted values can now be obtained.

From Eq. (1) the predicted collapse load would be P_c = 16.81 kips. Similarly, the theoretical shakedown load (Eq. 4) is P_s = 13.72 kips. The theoretical ratio of P_c to P_s is reasonably large to compensate for possible experimental error, P_c/P_s = 122.40%. For alternating plasticity to be avoided, Eq. 5(a) must be satisfied thus:

$$146.5^{\text{in.kips}} \leqslant \Delta M_y = 159.66^{\text{in.kips}}$$

thereby warranting the section chosen and the test set-up.

In Figure 9 the load deflection curve is plotted for the proportional loading test P-2. The deflection values plotted were the means obtained from the values at the two load points. The criterion used for determining the experimental collapse load was the load at which the deflection commenced to increase at a faster rate. This load was obtained at the intersection of the elastic and plastic slope lines at 17.08 kips, as shown in Figure 9. Also plotted was the theoretical curve as predicted by simple plastic theory.

Massonnet[2] in his paper adopted as his method for determining the collapse load the condition that the total deflection is twice the elastic deflection. This corresponds in Figure 9 to the load 17.58 kips. Likewise, Roderick and Phillips[8] assumed stress concentration under a concentrated load and effective over a length equal to the depth of the beam, thereby defining collapse occurrence when yielding has penetrated through the full depth of the section at the distance half the depth of the beam from the load. Based on this method, the predicted value is 19.2 kips. Figure 10 shows these values for test P-1.

It has been shown by various investigators[8] that when the bending moment distribution has a single maximum value under a concentrated load, the cross section at this point is capable of developing a greater resistance than the plastic moment. This behavior of increasing load as the beam deflects rapidly was manifested in the tests P-2 and P-1 as shown in Figures 9 and 10. The investigation of the actual stress distribution under the load which is rather complex is beyond the scope of this report. As observed in this investigation by the flaking of whitewash, yielding did not start directly under the loading points or the central support but rather on both sides of these points. It is also possible that strain hardening effects the increased carrying capacity of such sections under a concentrated load.

From the observed values of the end reactions, the moment distribution of the structure was determined. The moments at joints 2 and 3 were plotted against the applied load in Figure 11 for test P-2. Theoretically, when both moments at these two points attain the full plastic moment of the section, the structure collapses. As shown in Figure 11, the moments M_2 and M_3 became nearly equal in magnitude at load 16.0 kips. In test P-1 (Figure 12) M_2 and M_3 came close to each other at load 18.0 kips. The full plastic moment of the section based on the average lower yield stress of the tension coupons (Table 1) was 121.0 in-kips.

The observed moments were in agreement with the predicted values in the elastic range of the tests as shown by the theoretical elastic lines in Figures 11 and 12.

Test P-2 shown in Figure 11 was completely unloaded at load 17.7 kips and indicated the presence of residual moments under zero loads. A treatment on residual moments is contained in a latter part of this report. When the beam was loaded again till 20.06 kips, the phenomena that "metals, such as mild steel, have a memory" became evident as may be seen from the graph.

As mentioned previously, average rotation measurements were taken on a length equal to the depth of the beam. The average moment over the indicator gage length was computed from the observed moments at the joints, designated as M'. Shown in Figures 13 and 14 are the M-Ø curves for joints 2 and 3 for both proportional loading tests. The idealized M-Ø curve shown was derived from the material and geometrical properties of the section. Again, the increased carrying capacity of the section is manifested as shown on the graphs.

The three cyclic tests are shown in Figure 15 with the number of cycles plotted against the average deflection readings taken at the end of step (e) of the cycle (Table 1). Test C-1 with P_s = 17.0 kips attained a deflection of one inch after 10 cycles. This test represents a case of typical progressive collapse (excessive deformations).

Test C-2 was initially started at the applied shakedown load of 14.75 kips and stabilized fairly well after 6 cycles. Subsequent increments of 250 pounds were then applied till the load P_s = 15.25 kips. For all these loads the structure stabilized after application of 6 cycles. The final load of 16.00 kips was deemed as a progressive collapse behavior.

Test C-3 was intended to check the behavior of test C-2. The conduct of the beam at the load of 14.0 kips justified test C-2. Thereupon, the sought critical shakedown load of 15.5 kips was applied. The average ϕ measurements for the three yielding joints at the end of each cycle are shown in Figure 17.

A comparison between the proportional load test P-2 and the three cyclic tests is made in Figure 16. It is apparent from the illustration that much lower loads applied in cycles produce similar deflections to higher proportional loads. In test C-1, P_s = 17.0 kips needed 18 cycles to accomplish a deflection of 1.21 inches. In test C-3, P_s = 15.5 kips required less than 24 cycles to reach the deflection effected at 18.30 kips of the proportional load test.

Shown in Figure 18 are the M-ϕ values for joint 3 in test C-3 taken after every step of the cycle. The moment at the joint was calculated from the observed reaction readings at the end supports. Up to cycle 8, the applied shakedown load P_s was 14.0 kips. From cycle 9 to 24 the load value was raised to 15.5 kips. The numbers corresponding to the plotted points denote the cycle number. Steps (b) and (d) of the cycle (Table 1) were interchanged arbitrarily after every two cycles. Also shown in the graph is the ϕ_y - value (Table 2) of the section.

Step (e) of the cycle corresponds to the stage at which maximum moment occurs at joint 3. The readings obtained at step (e) as shown in Figure 18 were all indicated by their corresponding cycle number. The points showing minimum moment for the joint were those of step (c) of the cycle.

Three readings were taken as the load was being raised proportionately to apply step (a) denoted by S_1 at the start of the cycle test. After the second cycle the structure stabilized at P_s = 14.0 kips as substantiated till cycle 8. To commence the cycles for the desired experimental critical shakedown load of 15.5 kips, the corresponding point for step (a) is shown as S_9. Points 9b, 9c, 9d, and 9, denoted respectively steps (b), (c), (d) and (e) of the cycle. Indeed, points 9, 10b, 10c, 10d, and 10 show the successive steps of cycle 10. Cycle 19 was the critical stabilizing cycle. The following 5 cycles did not increase the rotation by any substantial amount. It is interesting to note that the beam stabilized only after a certain residual moment was being established. This was manifested by the readings at step (c) of the cycle -- the observed moment at joint 3 was approaching zero under a load of one kip at both load points (the elastic moment at such loading can be seen from Figure 21).

An attempt shall be made to show from experimental results the existence of residual moments necessary for continued deformation under cyclic loading. The steps (a), (b), and (c) respectively of the first cycle on test C-1 shall be considered.

Shown in Figure 19 are the observed moment distribution after loading step (a) and the computed corresponding elastic moment distribution on the structure. Crosshatched in the figure is the difference between elastic and observed moments. Complete unloading of the structure at this stage would occur in a purely elastic manner such that the elastic moments would be taken off and positive residual moments M_r would be left in the structure

(Figure 19). Therefore the actual moments can be thought of as being the sum of the elastic and residual moments.

Applying the load of step (b) as shown in Figure 20, the resulting positive elastic moment augmented by the previous positive residual moment at the now critical joint 4 would result in a value far beyond the available full plastic moment of the section. Hence plastic rotation of the joint took place. The observed moment distribution, after step (b) was applied, is also shown in Figure 20. Immediately below the figure is the new residual moment distribution now negative in sign. If the loading condition as shown in Figure 19 would follow, joint 3 would be affected and a further increase of the deflection at point 4 would result.

Step (c) of the cycle was applied next and verified the values and sign of the residual moments of step (b). This stage was nothing more than the unloading of step (b). It is illustrated in Figure 21. The observed moments were all negative in sign contrary to the elastic moment distribution for equal one kip loads. The negative residual moments agreed with that of Figure 20.

Summarized in Table 3 are the results of this investigation together with the various criteria previously mentioned for the determination of collapse loads. The observed collapse load based on deflection behavior was not far from the predicted value based on simple plastic theory. The Roderick-Phillips criterion may need further experimental studies when applied to statically indeterminate structures. The Massonnet Criterion is discretionary as was the criterion of the author. Definitely the collapse load is at least 10% greater than the critical shakedown load as applied to the chosen structure investigated.

Summary (Based on test results)

1. The theoretical collapse load can be relied upon when predicting plastic behavior of mild steel continuous beams under proportional loading.

2. A structure subjected to concentrated loads will carry a greater moment of resistance than the plastic moment. It is suggested that the actual stress distribution in the plastic range of the section immediately under a concentrated load be further investigated.

3. As predicted by theory, a statically indeterminate structure will deform continously resulting in excessive deflections when subjected to variable repeated loadings above a critical limit due to the existence of certain residual moments as a consequence of plastic behavior.

4. The experimentally determined shakedown load $P_s = 15.5$ kips above which incremental collapse would occur is 13% above the theoretically predicted value $P_s = 13.72$ kips. On the other hand it is still 8% below the theoretical collapse load $P_c = 16.81$ kips for proportional loading.

5. The cyclic tests proved that statically indeterminate structures will deflect successively under each cycle at loads considerably below proportionally applied loads causing equal deflections (See Figure 16).

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Table 1

Test Program - 4WF13 Shape

Test No.	Loading Type	Loads	Remarks
P-1	Proportional	2-equal loads	See Fig. 1
P-2	Proportional	Single load	See Fig. 2
C-1	Cyclic*	Ps = 17.0^K	
C-2	Cyclic*	Ps = 14.75^K; 15.0^K; 15.25^K 16.0^K	
C-3	Cyclic*	Ps = 14.0^K ; 15.5^K	

* Loading Cycle:

Step (a)

Step (b)

Step (c)

Step (d)

Step (e) = Step (a)

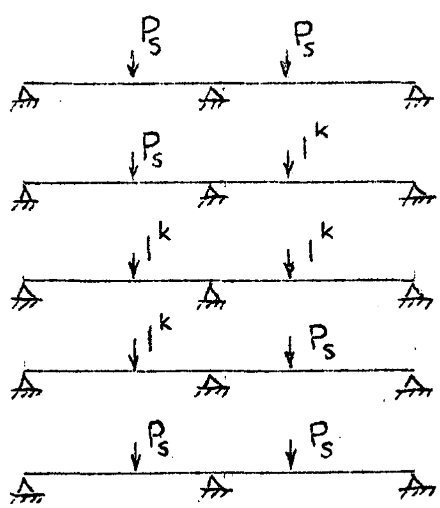


Table 2

Flange Tension Coupons

Yield Stress:	#T-1	:	σ_y = 39.30 ksi
	T-2	:	σ_y = 38.65 ksi
	T-3	:	σ_y = 37.40 ksi
	T-4	:	σ_y = 38.80 ksi

Material Properties:

Modulus of elasticity (assumed), psi	$E = 29.6 \times 10^6$
Yield stress (average of coupons), ksi	$\sigma_y = 38.5$

Geometrical Properties:

Flange width, in.	$b = 4.117$
Flange thickness (tapering) average, in.	$t = 0.366$
Depth, in.	$d = 3.808$
Web thickness, in.	$w = 0.2906$
Section modulus (weak axis), in.^{3}	$S = 2.073$
Plastic modulus (weak axis), in.^{3}	$Z = 3.143$
Moment of inertia (weak axis), in.^{4}	$I = 4.269$
Shape factor	$f = 1.516$

Section Properties of Shape:

Yield moment, in-kips	$M_y = 79.83$
Full plastic moment, in-kips	$M_p = 121.0$
Curvature at initial yield, rad/in	$\phi_y = \frac{M_y}{EI} = 0.63 \times 10^{-3}$
Available elastic moment range, in-kips	$M_y = 2M_y = 159.6$

Table 3

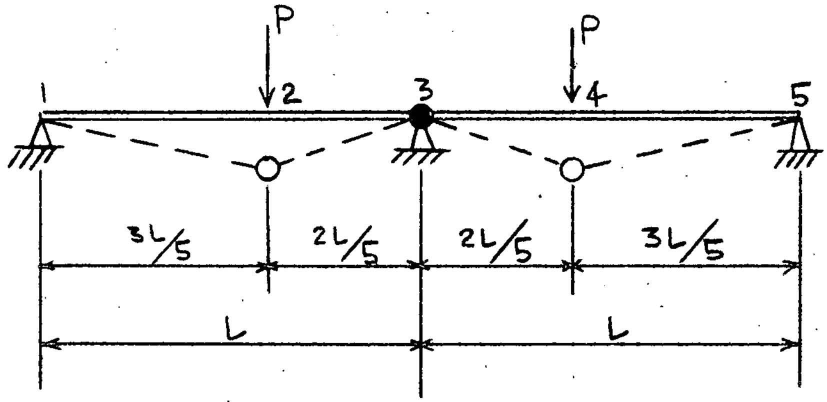
Summary of Test Results

1. Collapse Loads, Ps (kips):

Criterion	Massonet[2]	Roderick and Phillips (8)	Simple Plastic Theory (6)	Observed Values	
				Deflec.	Moments
Test P-2	17.58	19.20	16.81	17.08	16.00
Test P-1	17.58	19.20	16.81	17.68	18.00

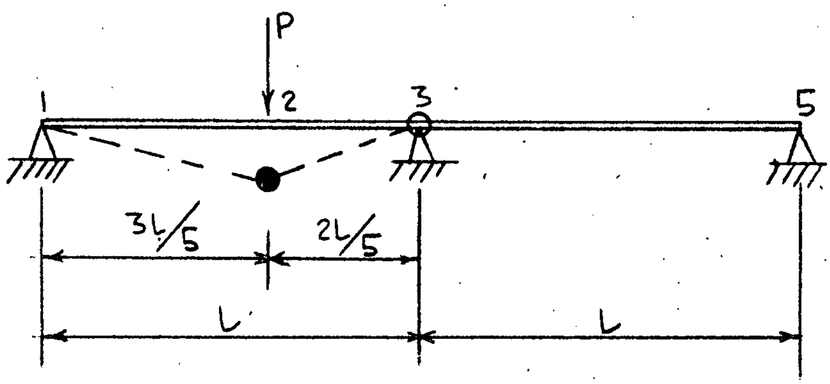
2. Critical Shakedown Load, P_s (kips)

	P_s Shakedown Load	P_c Collapse Load	P_c/P_s
Simple Plastic Theory	13.72	16.81	122.4%
Observed (from deflection criterion)	15.50	17.08	110.2%



CASE (a) - PROPORTIONAL LOADING w/ TWO LOADS

FIG. 1



CASE (b) - PROPORTIONAL LOADING w/ SINGLE LOAD

FIG. 2

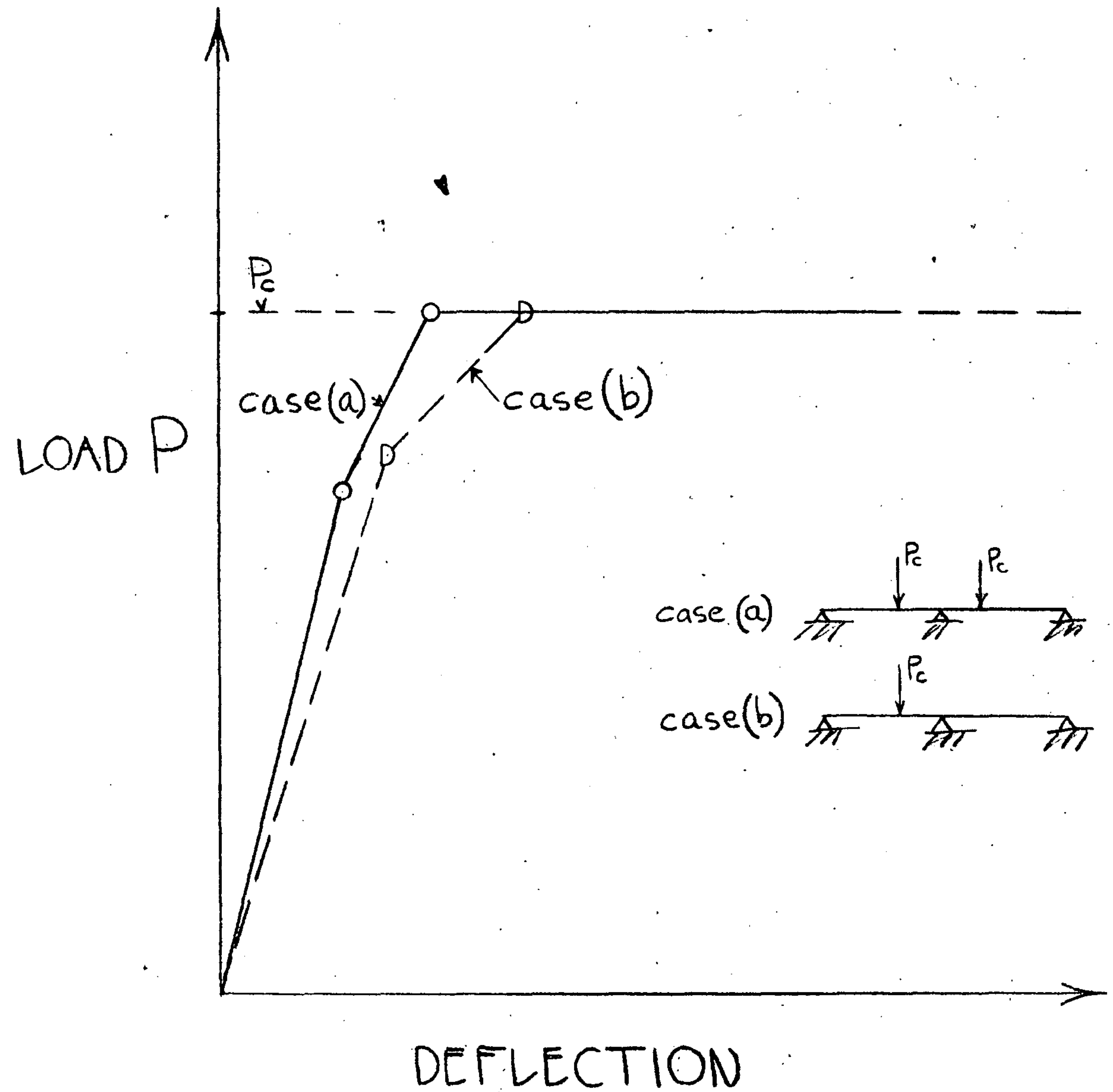


FIG. 3

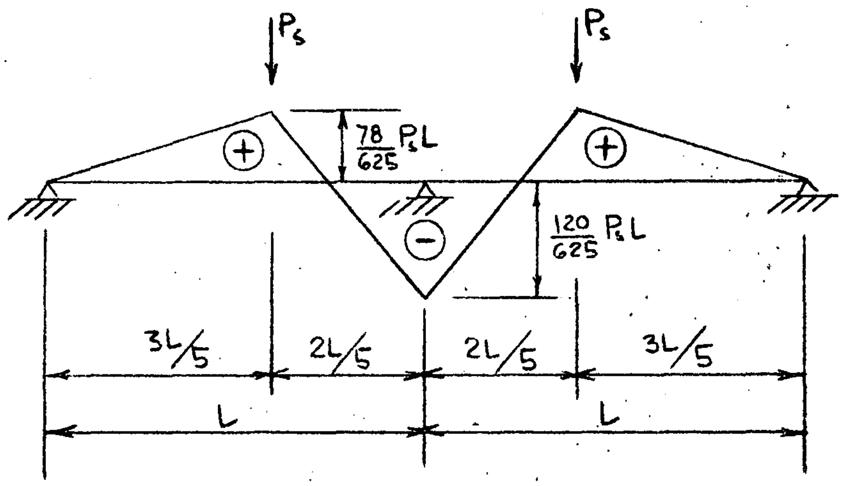


FIG. 4a

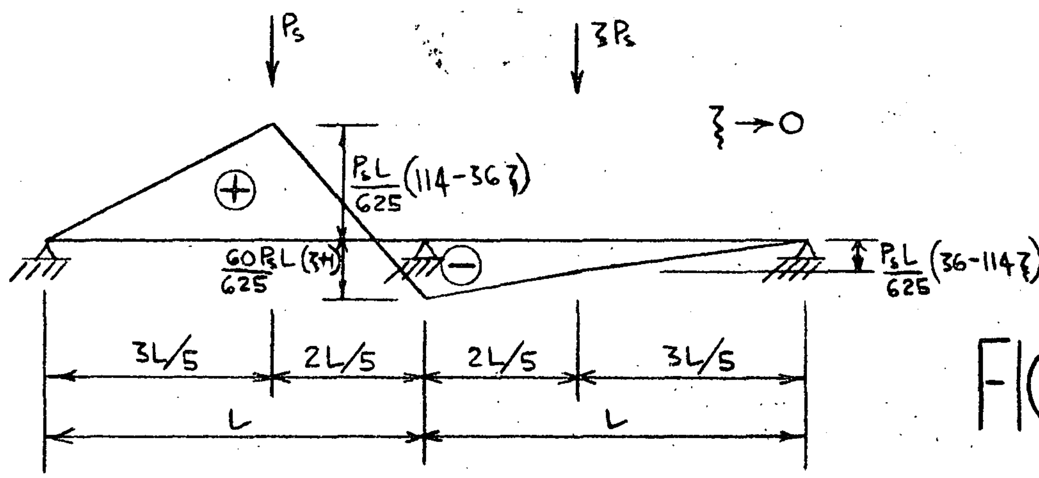


FIG. 4b

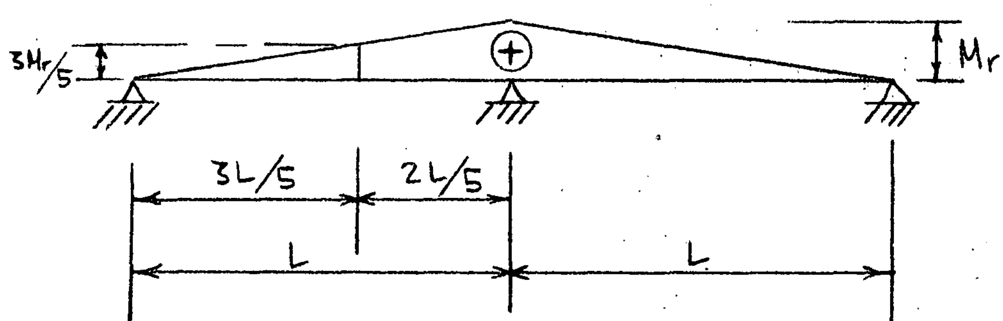


FIG. 4c

Figure 5(a). <u>General View of Test Set-Up</u>

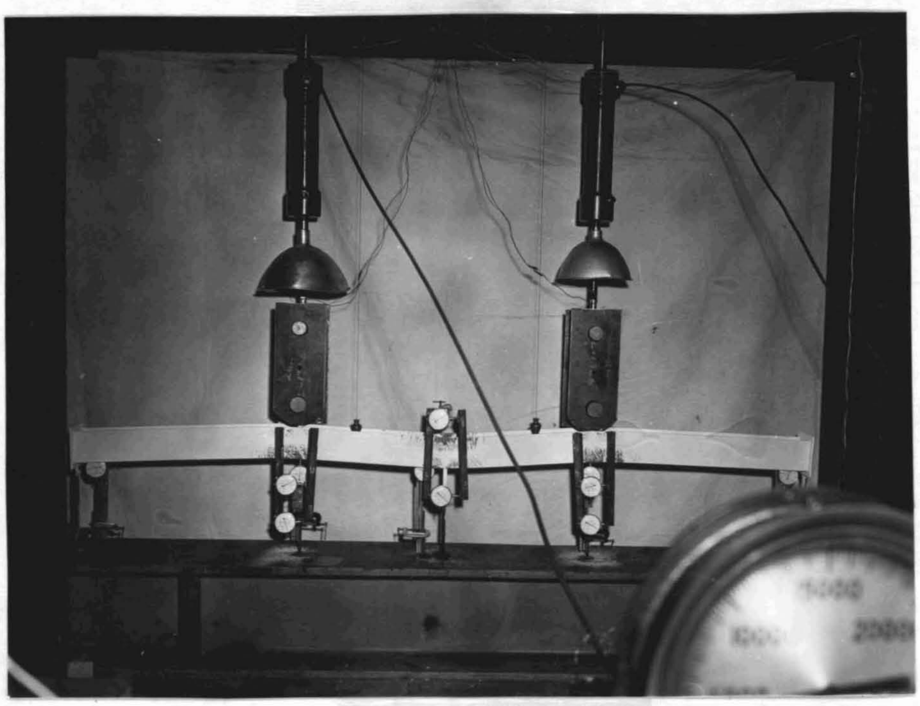


Figure 5(b). <u>Test Specimen</u>

Figure 6. End Support Detail Showing SR-4 Gages for Measuring Reaction

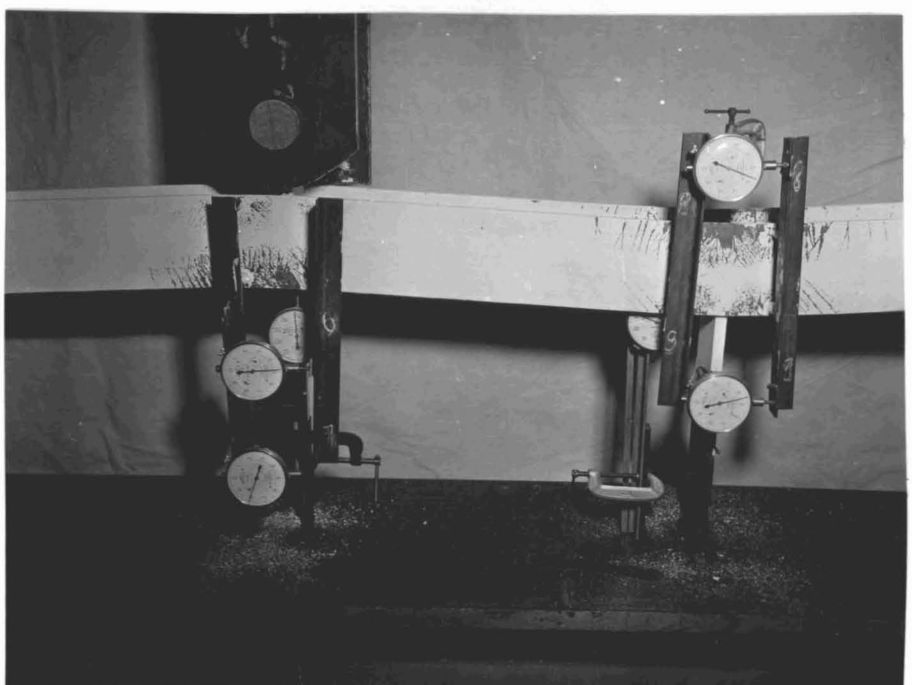


Figure 7. Details Showing Load Point and Central Support

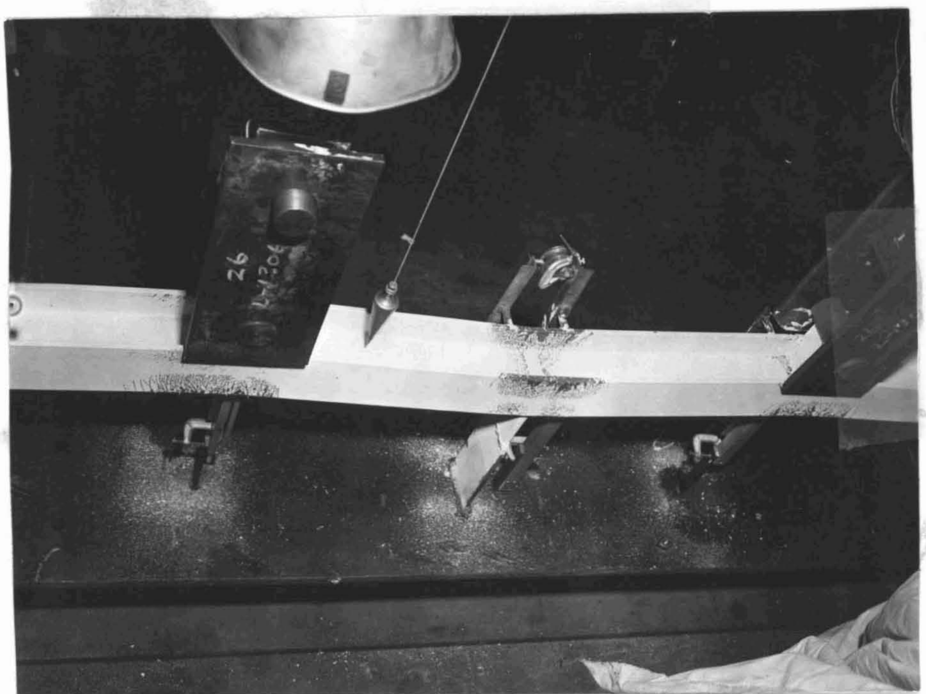


Figure 8. Extent of Yield at the Plastic Hinges

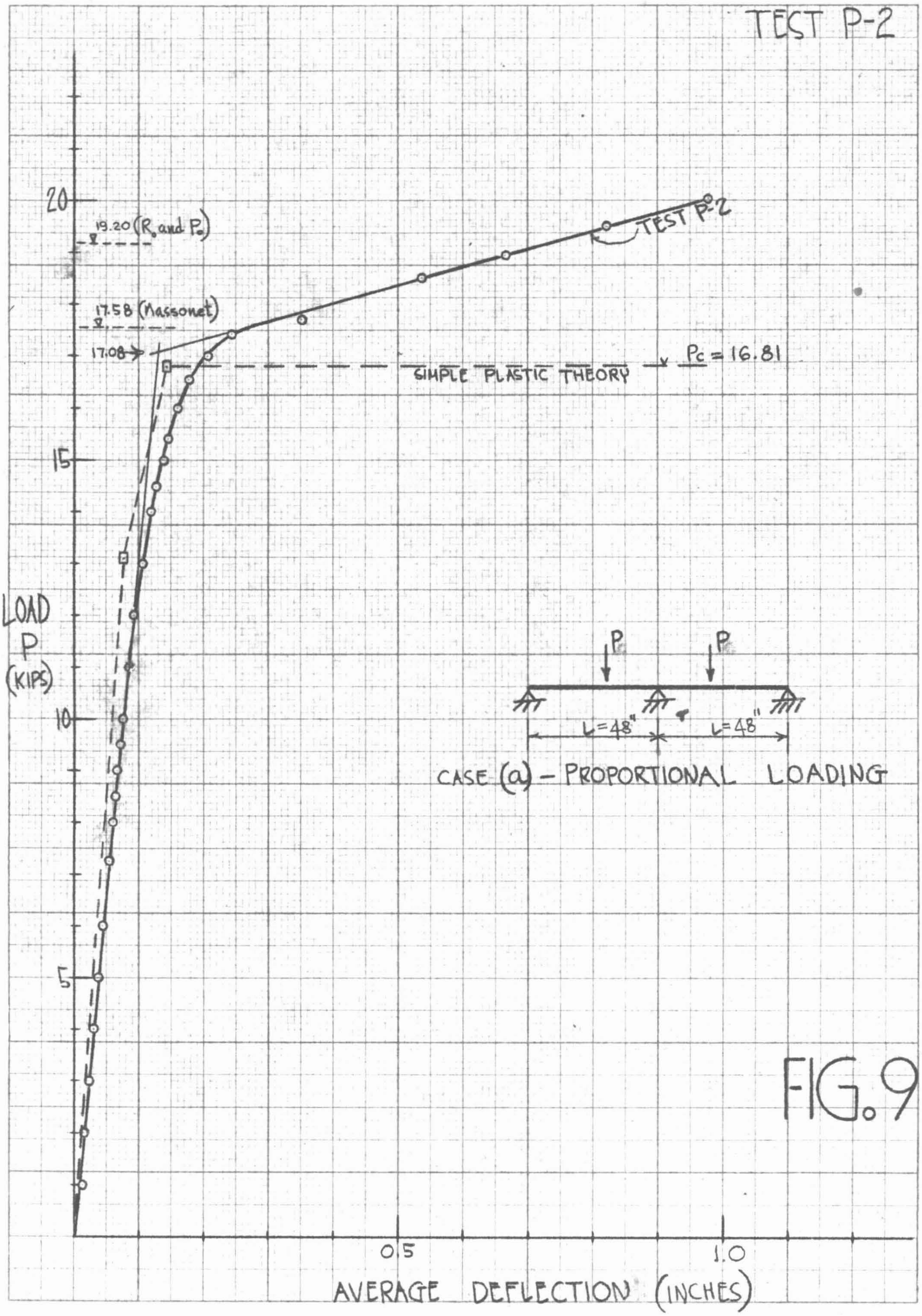
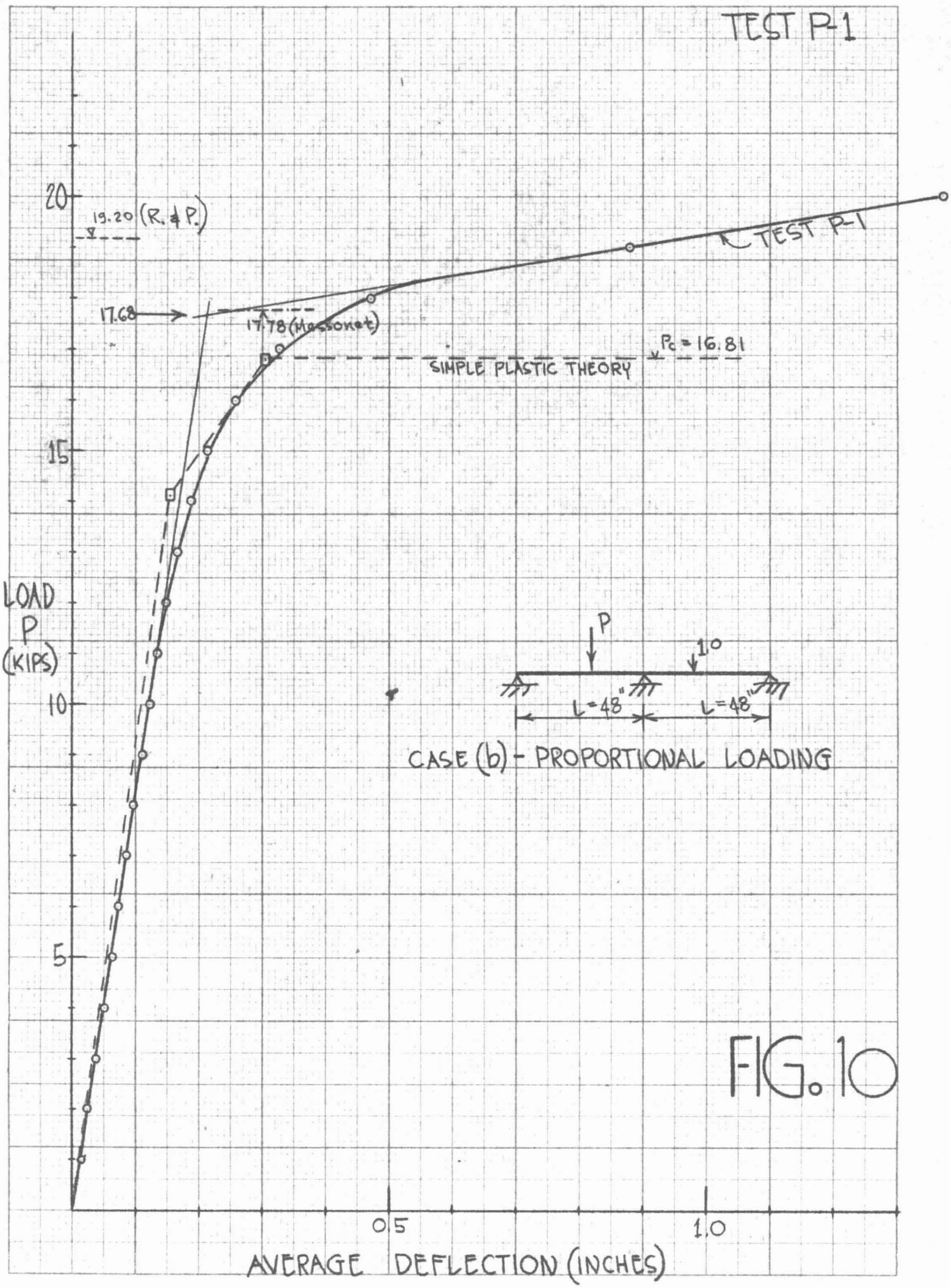

TEST P-2
20
19.20 (R, and P$_0$)
17.58 (Massonet)
17.08
TEST P-2
P_c = 16.81
SIMPLE PLASTIC THEORY
15
LOAD
P
(KIPS)
10
5
P
P
L=48"
L=48"
CASE (a) - PROPORTIONAL LOADING
0.5
1.0
AVERAGE DEFLECTION (INCHES)

FIG. 9



TEST P-1
19.20 (R. & P.)
17.68
17.78 (Massonet)
Pc = 16.81
SIMPLE PLASTIC THEORY
TEST P-1
LOAD P (KIPS)
20
15
10
5
P
1.0
L = 48"
L = 48"
CASE (b) - PROPORTIONAL LOADING
0.5
1.0
AVERAGE DEFLECTION (INCHES)

FIG. 10

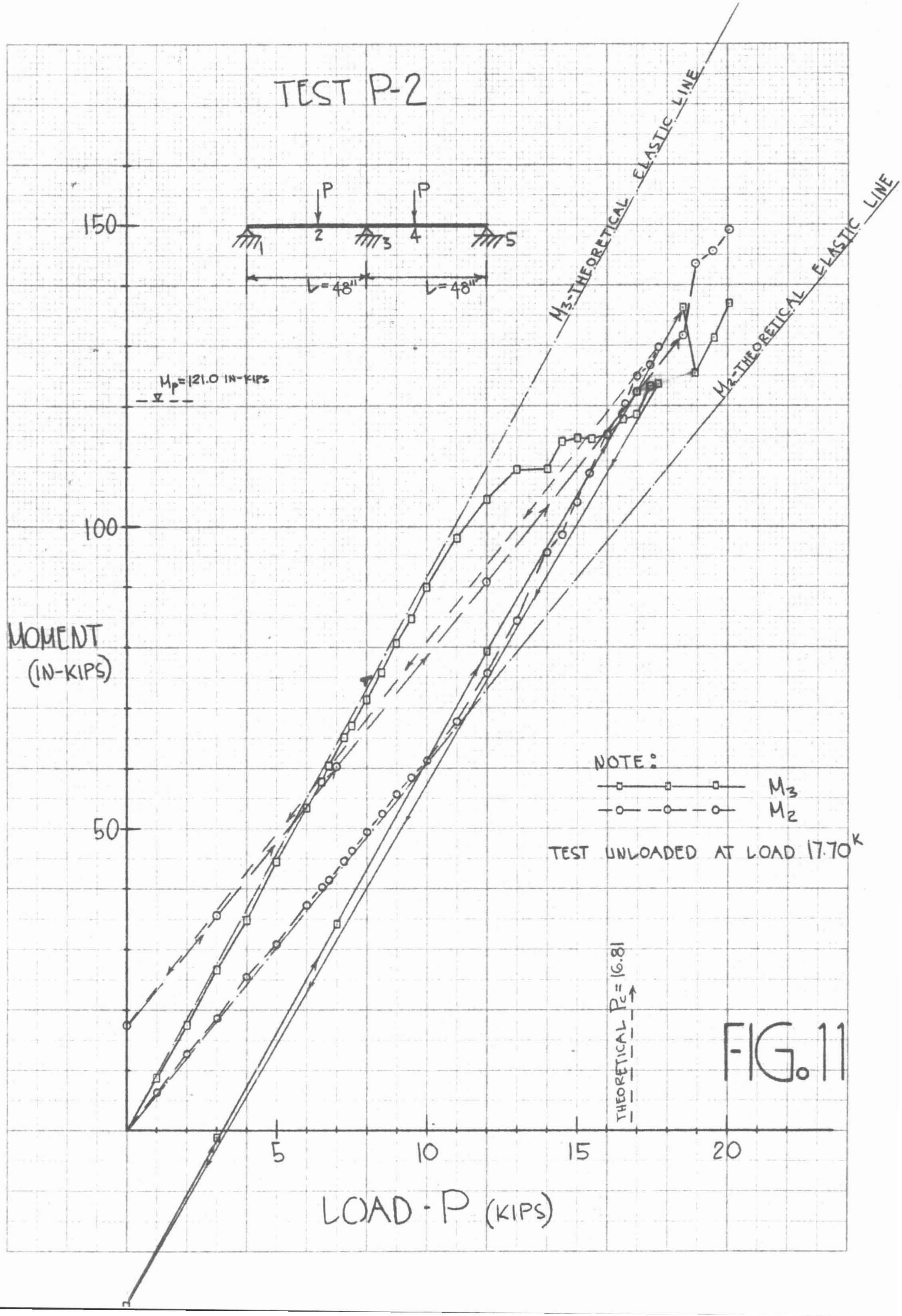
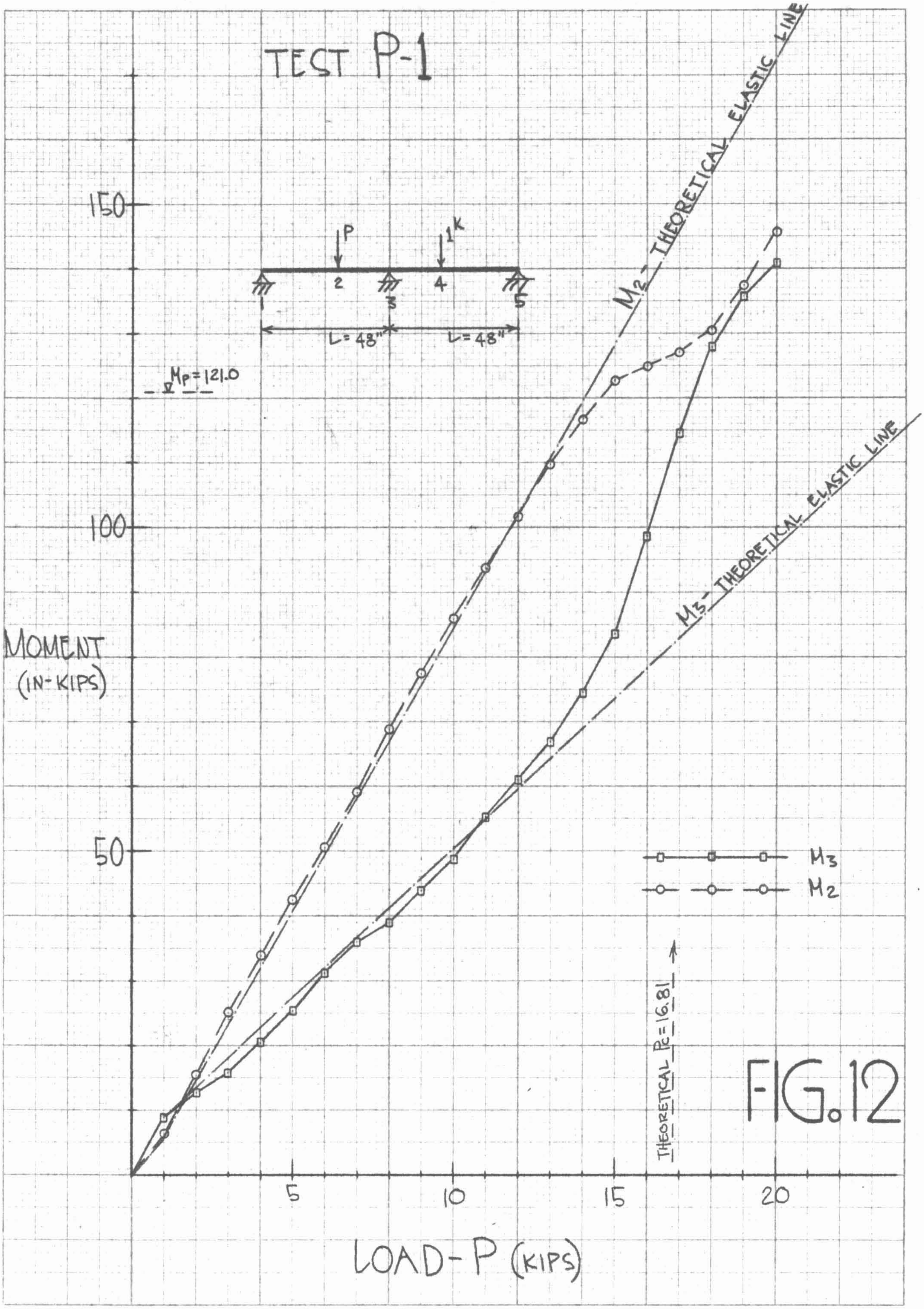

TEST P-2
P
P
1
2
3
4
5
L=48"
L=48"
M_p=121.0 IN-KIPS
150
100
50
MOMENT
(IN-KIPS)
M_3-THEORETICAL ELASTIC LINE
M_2-THEORETICAL ELASTIC LINE
NOTE:
M_3
M_2
TEST UNLOADED AT LOAD 17.70^K
THEORETICAL P_c = 16.81
5
10
15
20
LOAD - P (KIPS)

FIG. 11



TEST P-1
P
1^k
1
2
3
4
5
L = 48"
L = 48"
M_p = 121.0
150
100
50
MOMENT (IN-KIPS)
M_2 - THEORETICAL ELASTIC LINE
M_3 - THEORETICAL ELASTIC LINE
M_3
M_2
THEORETICAL P_C = 16.81
5
10
15
20
LOAD - P (KIPS)

FIG. 12

TEST P-2

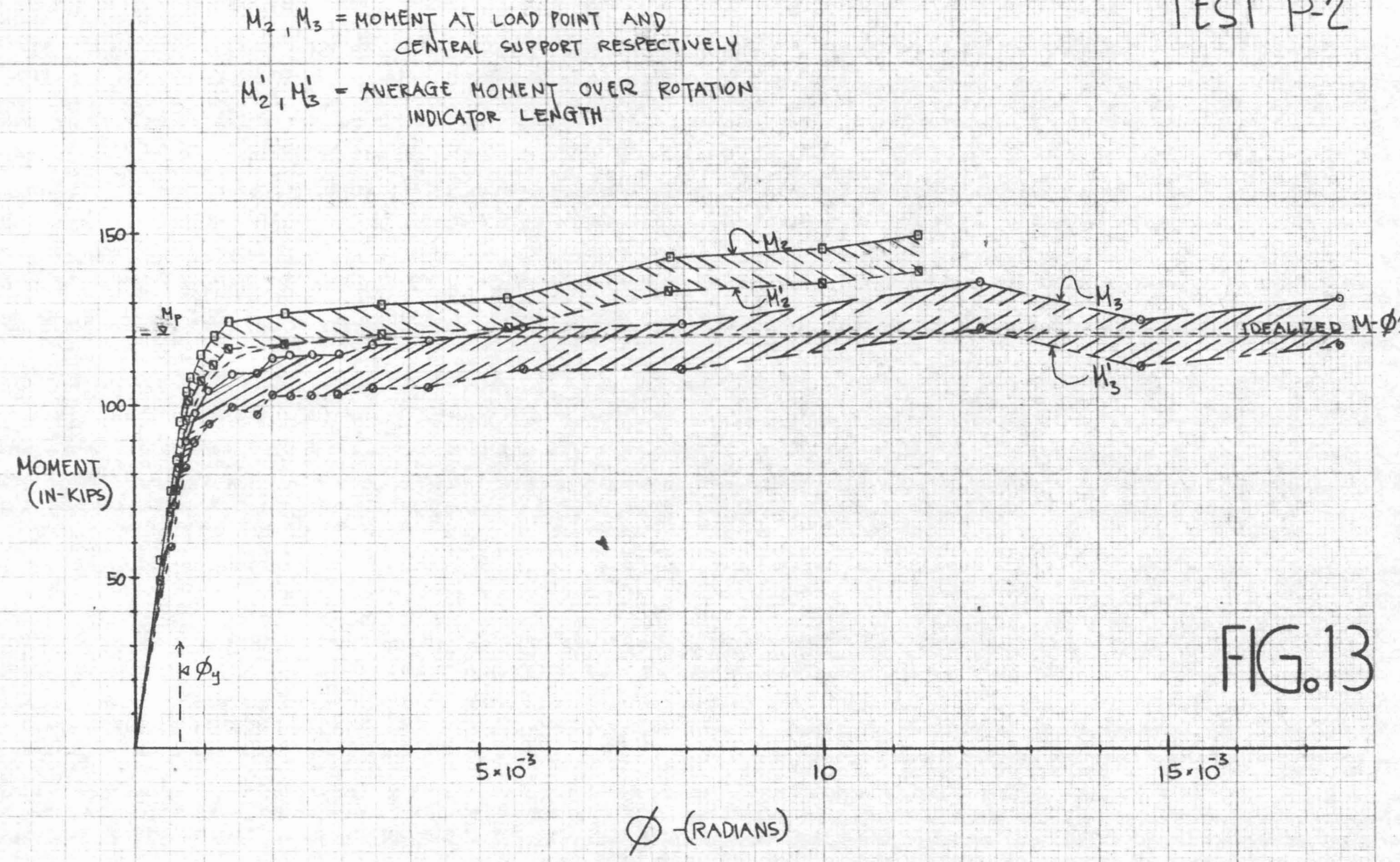


FIG. 13

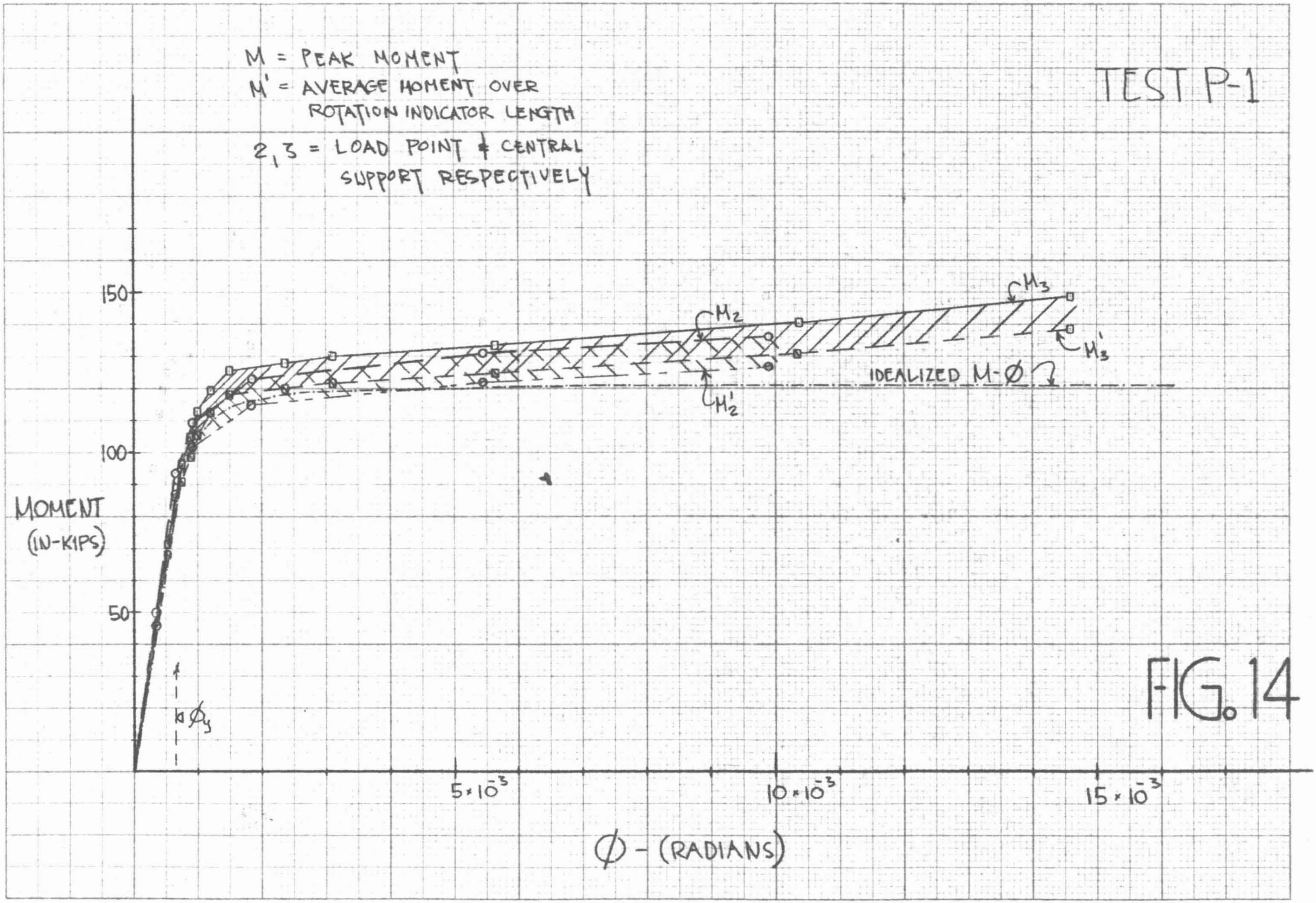

M = PEAK MOMENT
M' = AVERAGE MOMENT OVER ROTATION INDICATOR LENGTH
2, 3 = LOAD POINT & CENTRAL SUPPORT RESPECTIVELY
TEST P-1
150
100
50
MOMENT (IN-KIPS)
M2
M3
M'2
M'3
IDEALIZED M-Ø
Øy
5×10⁻³
10×10⁻³
15×10⁻³
Ø - (RADIANS)

FIG. 14

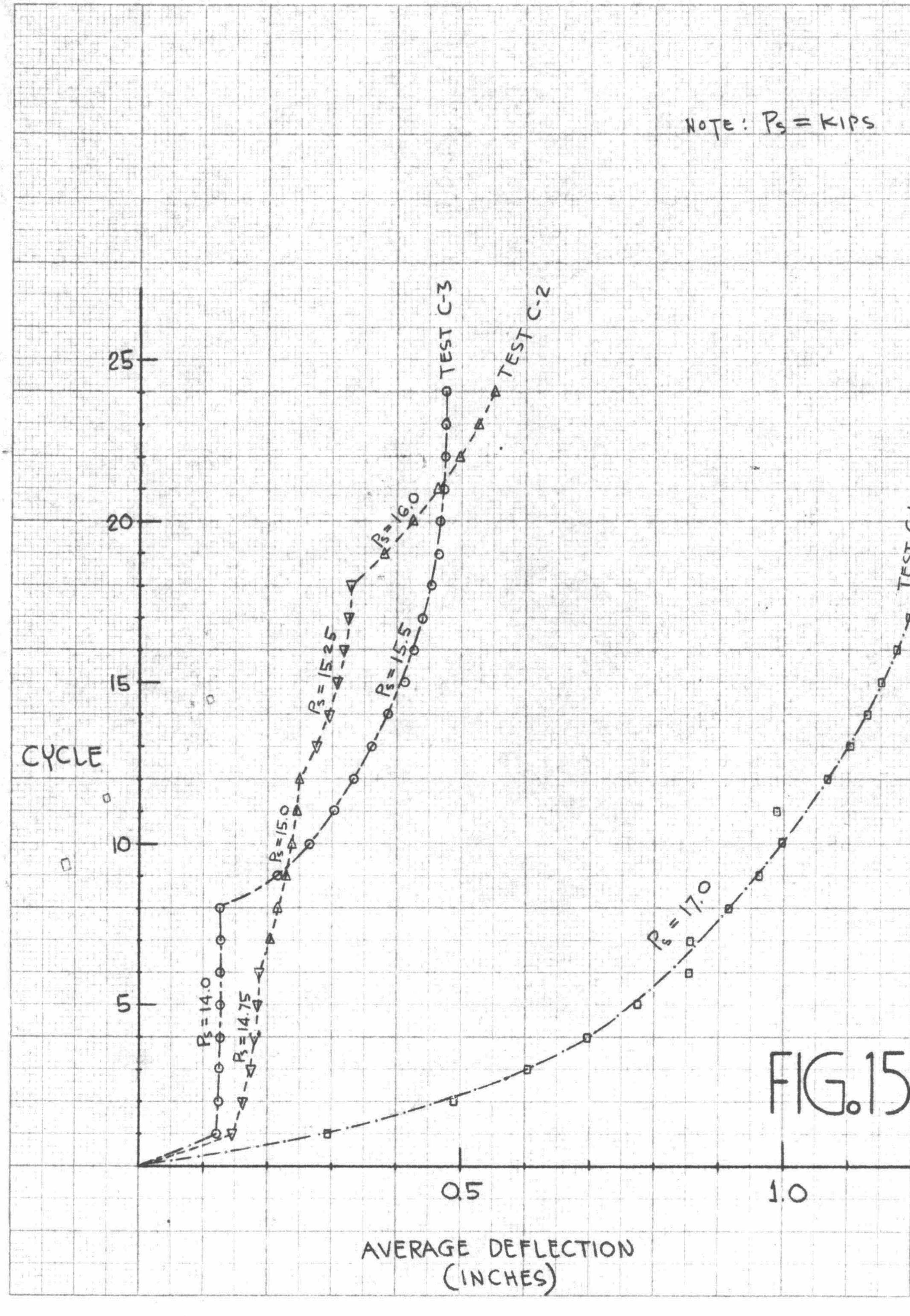
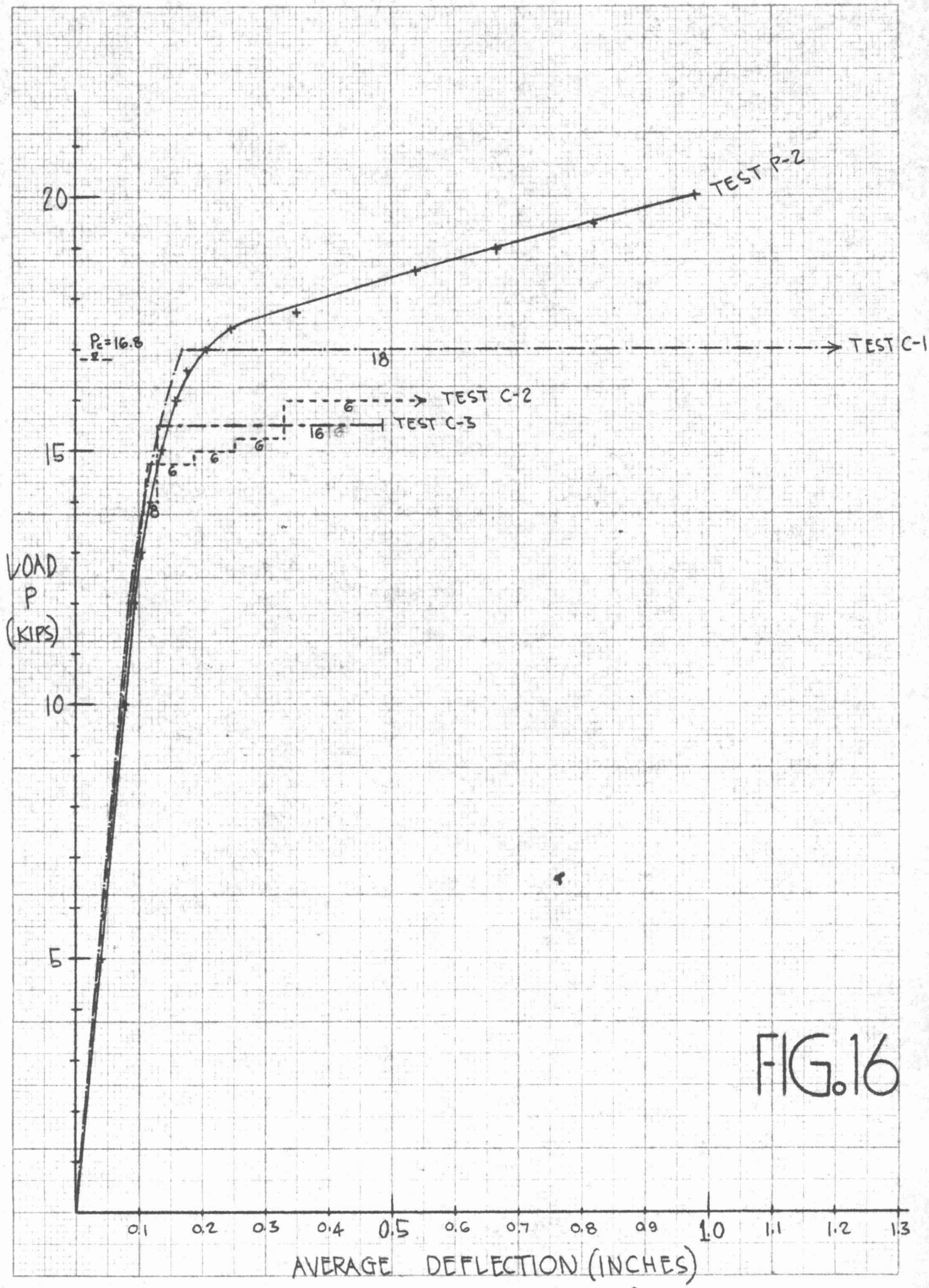

NOTE: P_s = KIPS
TEST C-3
TEST C-2
TEST C-1
P_s = 14.0
P_s = 14.75
P_s = 15.0
P_s = 15.25
P_s = 15.5
P_s = 16.0
P_s = 17.0
CYCLE
5
10
15
20
25
0.5
1.0
AVERAGE DEFLECTION
(INCHES)

FIG.15



TEST P-2
TEST C-1
TEST C-2
TEST C-3
$P_c = 16.8$
18
16
6
20
15
10
5
LOAD P (KIPS)
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
AVERAGE DEFLECTION (INCHES)

FIG. 16

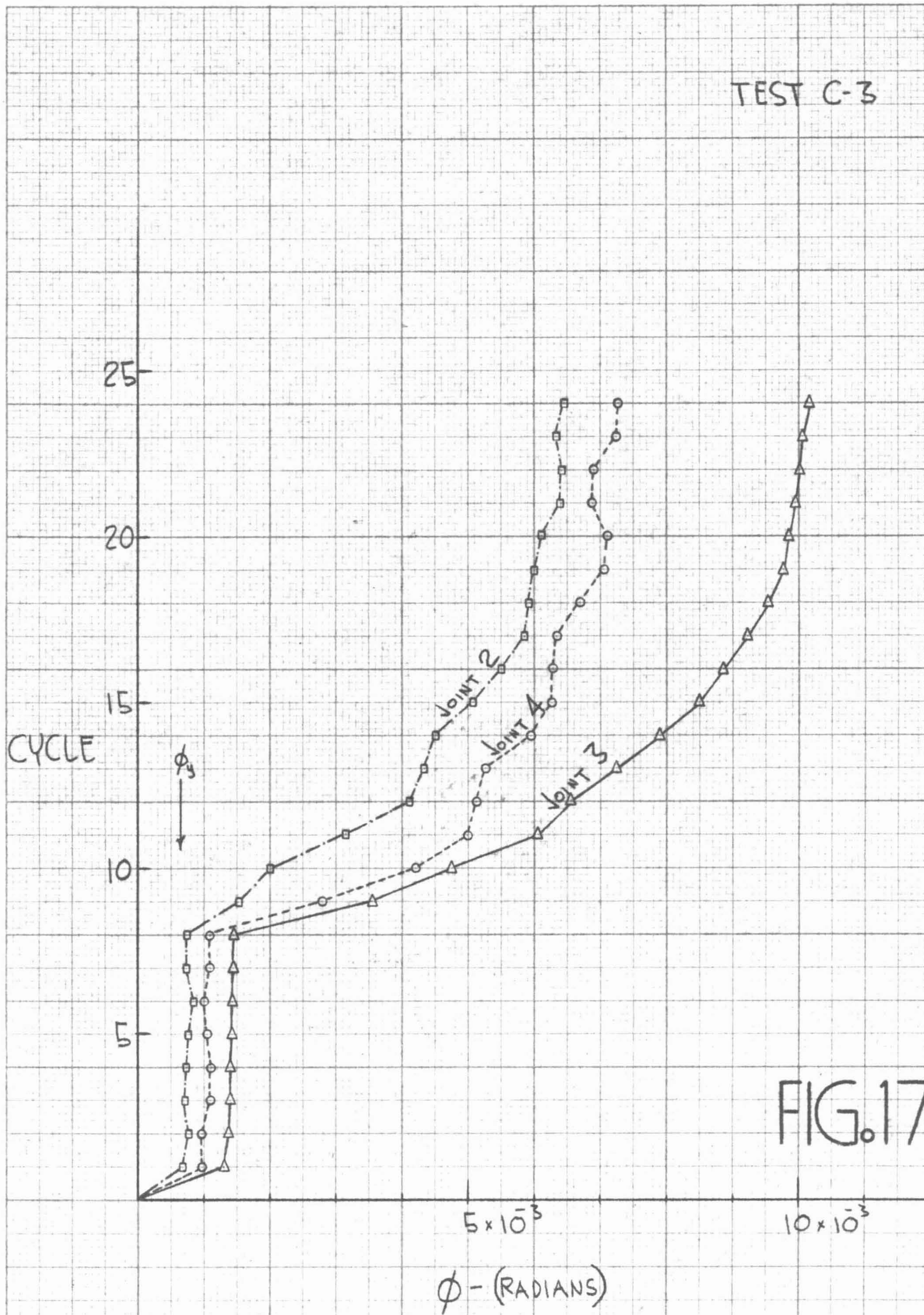

TEST C-3
CYCLE
25
20
15
10
5
ϕ_y
JOINT 2
JOINT 4
JOINT 3
5×10^{-3}
10×10^{-3}
ϕ - (RADIANS)

FIG.17

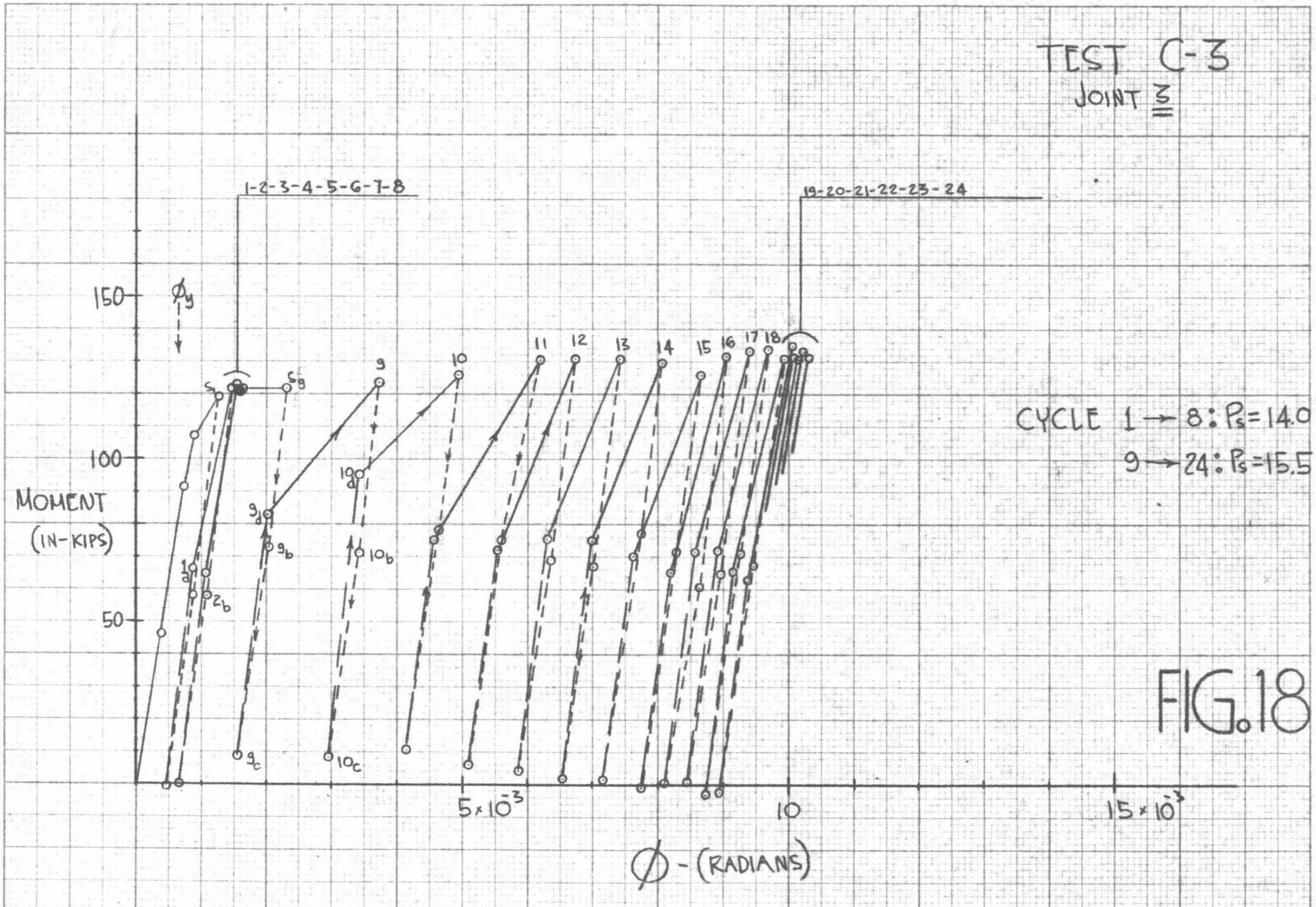

TEST C-3
JOINT III
1-2-3-4-5-6-7-8
19-20-21-22-23-24
ϕ_y
150
100
50
MOMENT
(IN-KIPS)
5×10^{-3}
10
15×10^{-3}
ϕ - (RADIANS)
CYCLE 1 → 8 : P_s = 14.0
9 → 24 : P_s = 15.5
9
10
11
12
13
14
15
16
17
18
9_d
9_b
9_c
10_d
10_b
10_c
2_b

FIG. 18

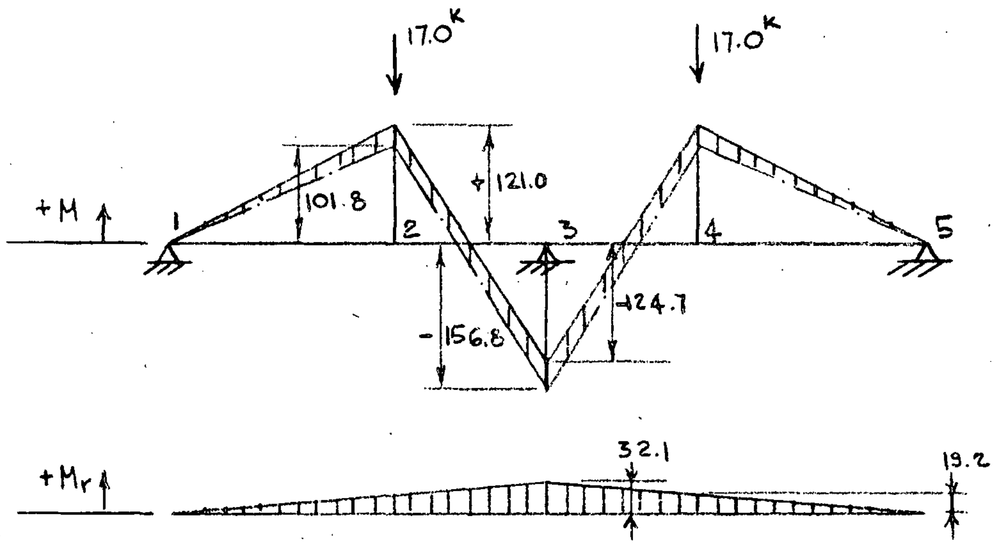


FIG. 19

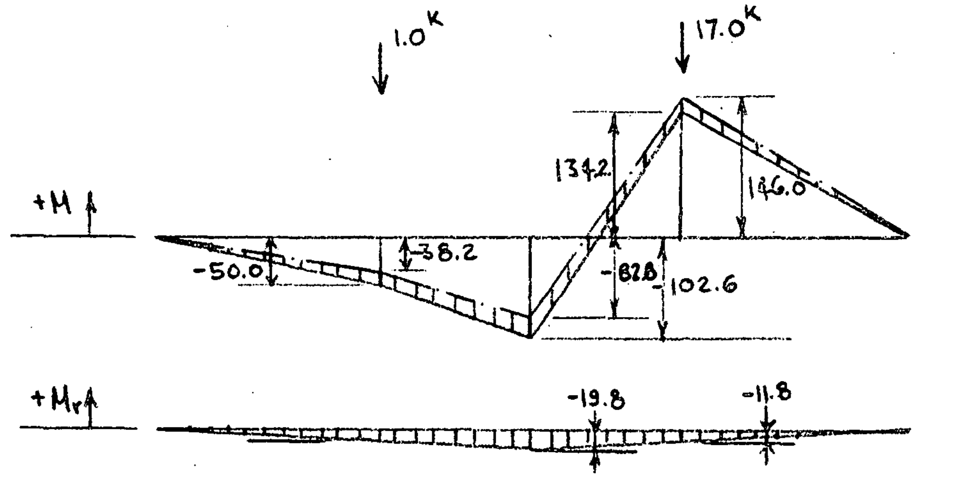


FIG. 20

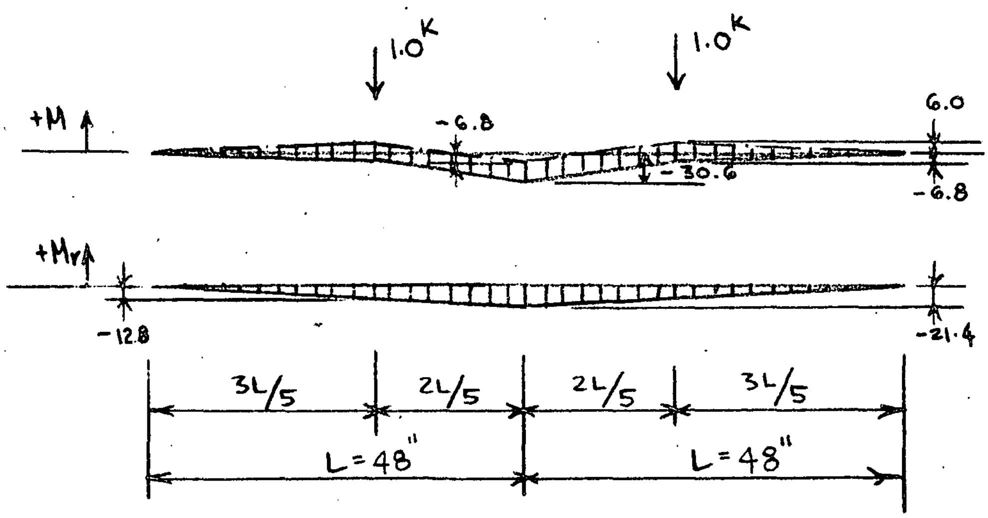


FIG. 21

NOTE:

ELASTIC MOMENT (IN-KIPS)

OBSERVED MOMENT (IN-KIPS)

L = 48 INCHES